suggested that the gas mixing results presented in this Note represent necessary but not sufficient conditions for lasing.

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Buckling Behavior of a Composite Beam Column

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AN analytical procedure was developed to predict the load vs deformation response of an axially loaded composite structure consisting of a column supported laterally by a beam. The analytical procedure was compared with an experiment which demonstrated the validity of the analysis and the existence of an interesting buckling phenomenon.

The composite beam-column system provides a means of controlling large deformations which occur when the critical load of a column is slightly exceeded. The load-deflection characteristic of a composite beam-column system would somewhat resemble that of a material tested in tension or compression rather than the characteristic column buckling curve. Thus, the composite structure system when subjected to axial loading, effectively exhibits a yield point and an ultimate strength as would a ductile material loaded beyond the elastic region.

There are a number of advantages for using the two-stage beam-column system as a structural element. Columns can be designed with a factor of safety based on a pseudo yield point, the point of first-stage buckling. For such columns used in structures, first-stage buckling would be a positive method of indicating that allowable external load levels have been exceeded. In addition, once the first-stage occurs, the remaining load carrying capacity would be known with assurance.

The beam-column system has been applied as a structural

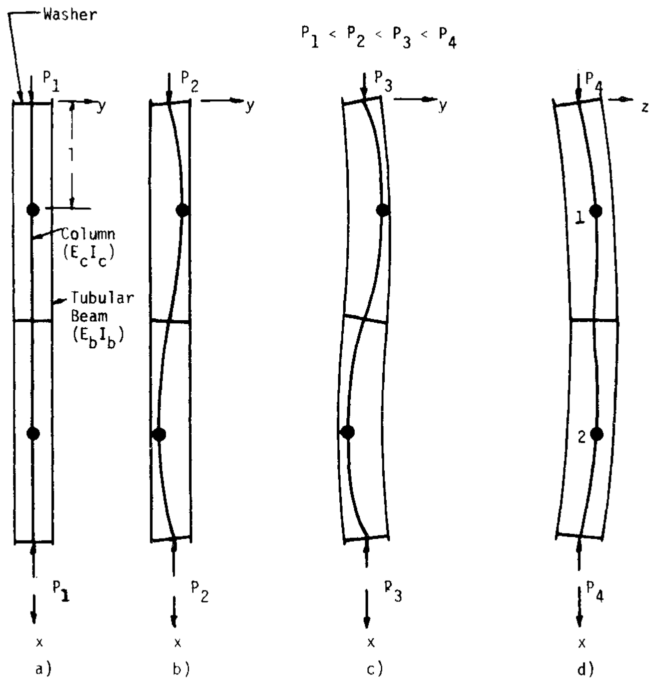

Fig. 1 Two-stage buckling of beam-column system.

element in a design, where in addition to axial load carrying capability, elongation under solar radiation was required to be a minimum. For this application, the beam component which provided lateral support to the column absorbed the thermal energy and expanded, whereas the column shaded by the beam experienced the required minimum elongation.

The beam-column system of Fig. 1a is such that the column will buckle in the second mode with a node at the midsupport point of the beam. The beam and column parameters are selected such that the beam remains straight until the first and third quarter points of the column make contact with the beam. The beam will begin to deflect in an antisymmetrical S-shape as the axial loading continues to increase, as shown in Fig. 1c.

The general solution for determining load vs end shortening for the composite structure was based on large deflection theory for the column and small deflection theory for the supporting beam. The system was assumed to remain in the *xy*-plane throughout the loading process. Figure 2 shows the load deflection relationships of the column from the time that the column buckles in the first mode. P_1 is the critical first-stage buckling load given by

$$\pi^2 E_c I_c / l^2 \qquad (1)$$

P_2 is the load required such that the column deflects through a distance y_1, where y_1 is the clearance between the beam and column at the first and third quarter points. The differential equation of the deflection curve for $P_1 \leqq P \leqq P_2$ is

$$E_c I_c (d\theta/ds) = -Py \qquad (2)$$

for $(0 < y(l) \leqq y_1)$. For values of $P > P_2$ the differential equation is

$$E_c I_c (d\theta/ds) + Py - Hx = 0 \qquad (3)$$

For a beam-column system with a given y_1, P_2 and l_2 can be found. Then setting a value of δ enables H to be determined. The general solution of the differential equation, which is similar to that given by Saelman,[1] can then be used to find an l_s and P consistent with $y(l) = y_1 + \delta$.

Assuming small deflections of the beam, H is given by

$$H = 3E_b I_b \delta / l_s^3 \qquad (4)$$

where l_s is the shortened length of the beam, and δ is the beam deflection.

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There is a constant β which has a minimum value of π^2 given by

$$\beta = 3E_b I_b / E_c I_c \qquad (5)$$

This minimum value is required in order that the column initially buckles in the second mode while the beam remains undeflected.[2]

Experimental work was performed in order to test the analytical solution. The model used for the experiment consisted of a steel column supported by a Plexiglass beam. The column was a threaded steel rod with rounded ends. Washers were assembled to the column by means of a nut on either side of each washer. The washers furnish the means of contact between the column and the beam and also kept the column concentric with the tubular beam. A Plexiglass beam was selected in order to be able to observe column as well as beam behavior. The column assembly having the outside diameter of its washers slightly less than the inside diameter of the Plexiglass tubular beam is shown as a complete beam-column assembly in Fig. 1a. It is assumed that no moment constraint exists between the washers and the beam.

The column used in the experiments was a number 8-32 steel rod with an over-all length of 15.50 in. The plexiglass tube with an i.d. of 0.50 in. and o.d. of 0.625 in. was selected such that the ratio of the beam to column flexural rigidity would be greater than 3.29. It is known that for a column with one equally spaced elastic support to behave as a rigid support, and buckle in the second mode, the flexural rigidity of the beam $E_b I_b \geqq 3.29 E_c I_c$ (Ref. 2). The actual ratio for the experimental model was 5.76.

The experimental result of the beam-column system having a clearance of 0.17 in. at the quarter points is given in Fig. 3, along with the theoretical curve. On the theoretical curve point *b* corresponds to the first stage critical buckling load of 57 lb. Curves *b-c* and *c-d* are found using the differential equations (2) and (3), respectively. For the model, first stage buckling started at point *b'* under a load of 53 lb. An additional amount of end shortening was required to bring the experimental load up to the theoretical load of 57 lb. Failure of the model to achieve the idealized point *b* and *c* is attributed to small inaccuracies in the column. At point *c'* on the experimental curve one of the washers first makes contact with the beam. The load increases until both washers make contact at point *e'*. As increased loading was applied to the model the experimental data followed the theoretical curve *c-d* quite well until reaching point *g'*. When the load of 91 lb, corresponding to point *g'*, was reached, the deformation mode was observed to change. Prior to this point all deformation took place in the *xy*-plane of Fig. 1. However, beginning at point *g'*, the washer contact points 1 and 2 were observed to move from the *xy*-plane, with 1 moving counterclockwise and 2 moving clockwise, as viewed along the *x*-axis. Thus, the column deformation mode is altered so that the column and beam tend toward a deformation mode lying in the *xz*-plane. It may be noted that the buckling load corresponding to a half-sine buckling mode of the composite structure is 96 lb, as shown by the dashed line in Fig. 3. Apparently, the deformed column and the deformed beam system loaded at point *g'* are altered with further increase in load in such a way that the column and beam tend to assume a half-sine mode in the *xz*-plane. The initiation of this transformation undoubtedly depends on the nature of the support points at locations 1 and 2 of Fig. 1. It is presumed that the maximum load for this composite structure cannot exceed the buckling load corresponding to the half-sine mode (the dashed line of Fig. 3) unless the center column is somehow restrained from deforming out of the *xy*-plane.

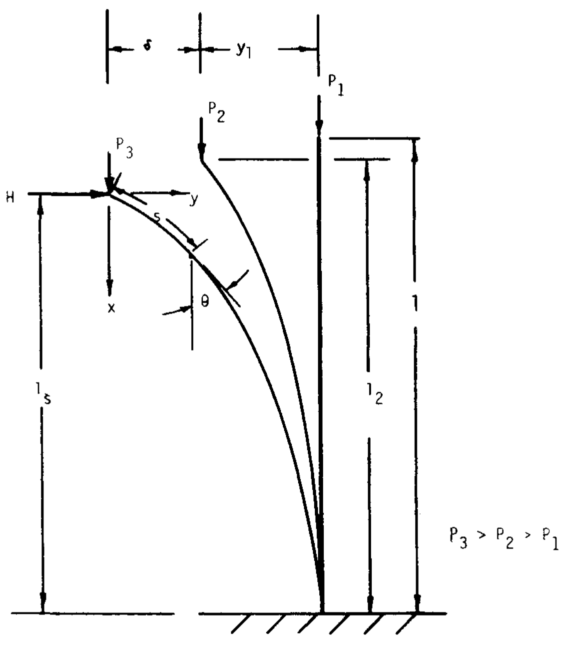

Fig. 2 Column behavior under increasing load.

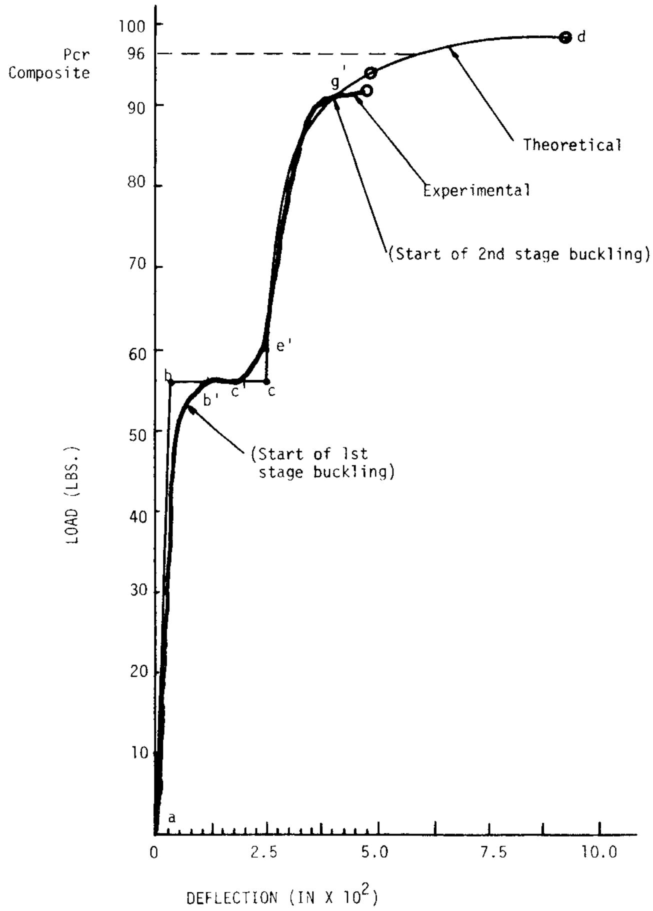

Fig. 3 Theoretical and experimental curves for two-stage buckling.

It is of further interest to note that it is possible to achieve loads higher than the 96 lb theoretical composite buckling load. An experiment was performed with five support washers instead of three. The composite structure was observed to buckle at a load of 100 lb. In this case, the beam deformed in the half-sine mode while the column was deformed so that it lay in the plane of the buckled beam. Lateral column displacement was in the direction of beam deflection, such that the column tended to contact the beam at four points midway between the five contacting washer supports.

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